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Technical Comments

Comment on "Offset Thrust Axes and Pitch Stability"

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IN a recent Technical Note, Solies[1] has considered the effect of thrust offset due to low- or high-mounted engines on the static longitudinal stability of an aircraft. At low speed and with no thrust effect the condition for static stability is that the pitch stability derivative $C_{m_\alpha} < 0$, which is achieved by positioning the c.g. ahead of the neutral point of the aircraft. Etkin[2] (p. 189) gives a more general criterion for static longitudinal stability derived from the constant term E in the characteristic quartic equation. Taking the derivative C_{z_u} as zero, this criterion reduces to

$$E = -C_{L_0}(2C_{m_\alpha}C_{L_0} + C_{m_u}C_{z_\alpha}) > 0 \tag{1}$$

where the subscript zero indicates the trimmed reference flight condition. Static stability then also depends on the speed stability derivative C_{m_u}. Since $C_{z_\alpha} < 0$, static stability is diminished if $C_{m_u} < 0$ with a divergent mode produced if $E < 0$. Etkin[2] gives an example that shows the effect of C_{m_α} and C_{m_u} on the time to double for the divergent mode.

Now consider the effect of thrust offset on the speed stability derivative C_{m_u}. At the trimmed reference flight condition the pitching moment coefficient due to the offset thrust is balanced by an aerodynamic pitching moment coefficient $C_{m_{\mathrm{aero}_0}}$, so that the pitching moment coefficient about the c.g. C_m is given by

$$C_m = C_{m_{\mathrm{aero}_0}} + \frac{T_0 z_T}{\frac{1}{2}\rho u_0^2 S\bar{c}} = 0 \tag{2}$$

where z_T is the distance of the thrust line below the c.g. The speed stability derivative C_{m_u} then has two terms associated with the variation in dynamic pressure and thrust with speed and is given by

$$C_{m_u} = u_0\left(\frac{\partial C_m}{\partial u}\right)_0 = \frac{\left(\dfrac{\partial T}{\partial u}\right)_0 z_T}{\frac{1}{2}\rho u_0 S\bar{c}} - \frac{2T_0 z_T}{\frac{1}{2}\rho u_0^2 S\bar{c}} \tag{3}$$

For turbojets thrust is almost constant with speed, whereas for propellers or turbofans thrust decreases with speed, i.e., $(\partial T/\partial u)_0 < 0$. A high thrust line for which $z_T < 0$ therefore gives $C_{m_u} > 0$ and enhances static longitudinal stability with the opposite effect produced by a low thrust line.

The conclusions made by Solies[1] appear to be based only on the effect of the variation of thrust with speed with no effect on stability predicted for constant thrust jet aircraft. The same incorrect conclusion is made by Raymer[3] (p. 429). To predict stability it is not necessary to make response calculations similar to that made by Solies.[1] Static stability or instability can be simply determined from the sign of E, which depends on C_{m_α} and C_{m_u} as given by Eq. (1). E can be expressed as

$$E = -2C_{L_0}^2 C_{z_\alpha}[(C_{m_\alpha}/C_{z_\alpha}) + (C_{m_u}/2C_{L_0})] \tag{4}$$

which in the case of a constant thrust turbojet and taking $C_{z_\alpha} \approx -C_{L_\alpha}$ reduces to

$$E = 2C_{L_0}^2 C_{L_\alpha}[-(C_{m_\alpha}/C_{L_\alpha}) - (T_0 z_T/mg\bar{c})] \tag{5}$$

The term within the brackets is precisely the stick-fixed static margin derived by Solies[4] with the condition $L = mg$ imposed and as determined from conventional flight tests. It follows that the condition for longitudinal stability is that the static margin with $L = mg$ is positive as opposed to the pitch stability condition $C_{m_\alpha} < 0$.

References

[1]Solies, U. P., "Offset Thrust Axes and Pitch Stability," *Journal of Aircraft*, Vol. 31, No. 5, 1994, pp. 1217–1219.
[2]Etkin, B., *Dynamics of Flight-Stability and Control*, 2nd ed., Wiley, New York, 1982.
[3]Raymer, D. P., *Aircraft Design: A Conceptual Approach*, 1st ed., AIAA Education Series, AIAA, Washington, DC, 1989.
[4]Solies, U. P., "Effects of Thrust Line Offset on Neutral Point Determination in Flight Testing," *Journal of Aircraft*, Vol. 31, No. 2, 1994, pp. 362–366.

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BLOY'S comments are helpful in clarifying the role of the speed stability derivative C_{m_u} in conjunction with offset thrust lines. Since he does not give a physical interpretation of the equations, I offer the following in support of his arguments.

The thrust moment $M_{T_0} = T_0 z_T$ of a constant thrust device does not change with angle of attack nor with velocity, and therefore does not directly affect longitudinal stability. This view was expressed by Raymer[1] (p. 429) and adopted in my articles.[2–4]

This view, however, did overlook the role of the aerodynamic moment,

$$M_{\mathrm{aero}_0} = C_{m_{\mathrm{aero}_0}}(\rho/2)u_0^2 S\bar{c} = -M_{T_0} \tag{1}$$

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which is required to balance the thrust moment of a vehicle in steady free flight. This moment is positive (pitchup) for high thrust lines ($z_T < 0$) and increases with speed (all else constant), providing speed stability. Conversely, a low thrust line causes a negative aerodynamic moment that is destabilizing.

This also explains the difference between pitch stability neutral points obtained from α sweeps in wind tunnels and stick-fixed neutral points obtained from conventional flight tests that rely on data taken at steady flight conditions at various airspeeds. The difference is often not known or misinterpreted, with the expectation being that the points should be identical and have the same significance,[4] or that discrepancies are solely caused by Reynolds number effects or other idiosyncracies. As I have shown,[2] flight tests of an aircraft with an offset thrust line will shift the wind-tunnel neutral point by an additional static margin

$$\Delta h_n = -(T/W \cos \gamma)(z_T/\bar{c}) \tag{2}$$

for jets [Ref. 2, Eq. (14)], and

$$\Delta h_n = -\tfrac{3}{2}(T/W \cos \gamma)(z_T/\bar{c}) \tag{3}$$

or

$$\Delta h_n = -\frac{3}{2}\, \eta_p P \sqrt{\frac{(\rho/2)SC_L}{(W \cos \gamma)^3}}\, \frac{z_T}{\bar{c}} \tag{4}$$

for props [Ref. 2, Eq. (20)].

As to the definition of static stability, Etkin[5] himself introduces longitudinal stability as pitch stability at constant speed (p. 13), which leads to the stability condition $C_{m_\alpha} < 0$ and the definition of a pitch stability neutral point h_n (p. 23). He emphasizes that "the trim-slope criterion obtained in flight test can be definitely misleading as to stability" (p. 25).

As Bloy points out, Etkin later introduces a more general definition of static longitudinal stability (p. 189), namely nondivergence, characterized by the factor $E > 0$ in the linearized equations of motion. E contains the speed stability term C_{m_u}, which relates directly to Δh_n.

Using this definition of longitudinal static stability, a high thrust line aircraft would indeed be stable for c.g. positions up to the stick-fixed neutral point $h_{nsf} = h_n + \Delta h_n$.

A warning must be issued, however, to not use this stick-fixed neutral point as the rear boundary for the aircraft's operational c.g. envelope. The additional static margin Δh_n varies with thrust, and the aircraft is unstable in pitch for $T = 0$ at c.g. locations $h > h_n$. If an engine failure occurs at such c.g. locations the pilot faces a vehicle that is not only out of trim, but also unstable in pitch. Unless artificial stability augmentation is used, the aft c.g. limit should be based on neutral points obtained in gliding flight, or in wind tunnels. If those are not available, Eqs. (2) or (4) may be used to calculate Δh_n.

References

[1]Raymer, D. P., *Aircraft Design: A Conceptual Approach*, 1st ed., AIAA Education Series, AIAA, Washington, DC, 1989.

[2]Solies, U. P., "Effects of Thrust Line Offset on Neutral Point Determination in Flight Testing," *Journal of Aircraft*, Vol. 31, No. 2, 1994, pp. 362–366.

[3]Solies, U. P., "Offset Thrust Axes and Pitch Stability," *Journal of Aircraft*, Vol. 31, No. 5, 1994, pp. 1217–1219.

[4]Solies, U. P., "Comment on Model Flight Tests and Neutral Point Determination," *Journal of Aircraft*, Vol. 31, No. 4, 1994, pp. 1007, 1008.

[5]Etkin, B., *Dynamics of Flight—Stability and Control*, 2nd ed., Wiley, New York, 1982.